Base Drag of a Thick Annular Jet

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A theoretical and experimental study is made of the base drag of a thick axisymmetric annular jet operating free from ground effect. An approximation for the recirculation mass flow in the base region is obtained through the use of an experimentally determined mixing coefficient. A thrust recovery factor, based on this mixing coefficient, is derived theoretically and compares favorably with the experimental results. These results show that the base drag losses of a thick annular jet decrease with increasing jet thickness to base diameter ratio. The loss in jet thrust is found to be less than 10% for the worst case considered (jet thickness to base diameter ratio of 0.048).

Nomenclature

A_B = area of base = $\pi D^2/4$
A_J = area of jet = $\pi t_i(t_i + D)$
C_{PB} = bubble pressure coefficient = $(p_\infty - p_B)/\rho u^2$
D = base diameter
I = jet impulse = $J + \int p \cdot da$
J = jet momentum flux
k = elliptic integral modulus
L = lift
$\dot{m}$ = mass flow rate
p = static pressure
R = radius of curvature
Re = Reynolds number
s = streamwise coordinate
t = jet thickness
u = jet streamwise velocity component
V = inlet pipe velocity
x = axial cordinate
y = normal coordinate
η = thrust recovery factor
θ = flow angle
ρ = density
ϕ = elliptic integral amplitude

Subscripts

B = bubble, base
c = jet centerline coordinates where the inner stream surface coalesces
i = initial conditions, nozzle exit
o = flow downward from the reattachment region
R = recirculation through the base region
∞ = conditions at infinity

Introduction

THE term "Base drag" is commonly used to describe the momentum losses associated with a flow around the base or after portion of rockets and plug nozzles. Such a flow is characterized by a closed recirculation region (bounded on the upstream side by the base) and a resulting negative pressure on the base which acts as a reduction in lift or thrust. The base drag concept can be extended to the recirculating flow common to the base region of peripheral or curtain jet VTOL and air cushion vehicles (ACV's) when operating out of ground effect.

In recent years, the annular jet has been the subject of numerous investigations in conjunction with the development of the modern ACV. The majority of these studies utilized a thin annular jet with the assumptions of constant jet curvature, constant recirculation bubble pressure and a "two dimensional" annular jet configuration.[1,2] The results of these studies are known to give reliable predictions for the performance of the thin annular jet vehicle when operating near the ground, but they fail to accurately predict the jet characteristics when the vehicle moves out of ground effect.

The two dimensional curved jet reattaching to an offset parallel plate has been investigated by Bourque and Newman[3] and Sawyer[4]. These studies were successful in establishing the growth characteristics of the "thin" curved jet and predicting its behavior. The term "thin" is usually taken to imply that the jet is fully developed for the major portion of its travel between the exit and the reattachment point (as described by Townsend[5]). The published results also include some pressure measurements showing the recirculation vortex and the reattachment point pressure rise. The two-dimensional theory for base drag and related phenomenon cannot be directly applied to a three-dimensional problem in which continuity demands that the jet have a variable thickness and a variable radius of curvature as it moves downstream and coalesces.

In this study, the jet is first assumed to be inviscid in obtaining a relationship between the recirculation bubble pressure and the jet curvature. Flow entrainment along the inner surface of the jet is then considered in a momentum balance to determine the thrust recovery factor as a function of jet mixing and jet geometry. The mixing coefficient is evaluated from experimentally determined reattachment lengths. No attempt was made to solve for the details of the jet flow development.

Theory

The axially symmetric jet configuration treated in this study is shown in Fig. 1. The following assumptions are made consistent with the scope of this study as well as previous work by other investigators on related flow configurations: 1) Incompressible and steady flow. 2) The base bubble pressure (p_B) is constant within the region bounded by the vehicle base and the jet inner stream boundary. 3) The pressure varies linearly across the jet from the base bubble at the inner surface, to atmospheric pressure at the outer jet surface. This linear approximation is assumed to apply from

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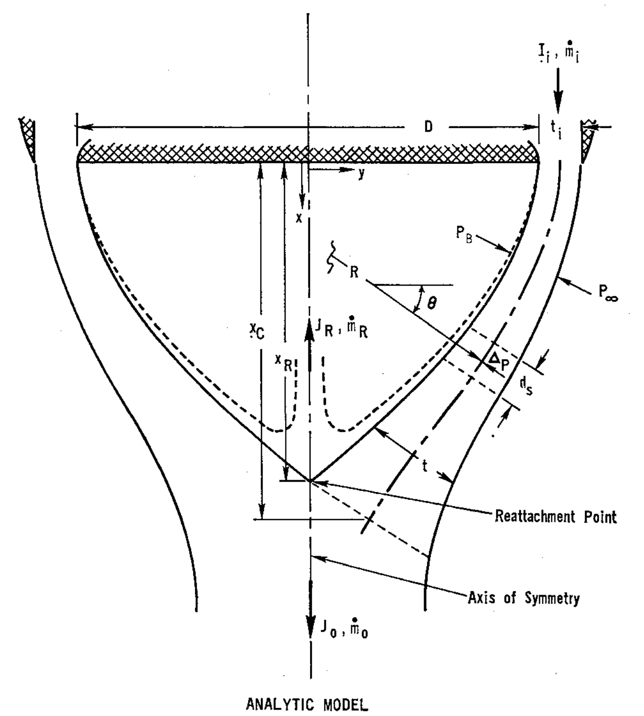

Fig. 1 Analytic model.

the jet exit up to the reattachment point. 4) The recirculating mass flow within the base region is much less than the initial jet mass flow.[6] 5) Jet momentum is conserved along the path of the reattaching jet.[6,7] 6) For the initial force balance (in determining appoximate jet curvature) the fluid is assumed to be inviscid. Thus, consistent with the other assumptions, the flow up to the reattachment point will have a constant velocity. 7) The flow velocities entering and leaving the reattachment region are equal.[1]

Position of Jet Reattachment Point

The reattachment point is taken as the point where the stream surface issuing from the inner edge of the jet exit would meet downstream at the vertex if there were no recirculation (Fig. 1). Assumption 4 allows the determination of the jet reattachment point position without accounting for the jet geometry changes associated with entrainment and recirculation within the base cavity. Considering a force balance on a flow element of the jet and evaluating the pressure force at the element centerline gives

$$2\pi y \cdot ds \cdot \Delta p = \dot{m} \cdot u d\theta \qquad (1)$$

Equating the mass flow from the nozzle to the mass flow at some downstream station determines the jet thickness (since u is constant)

$$t = t_i(t_{i+D})/2y$$

The pressure difference across the jet can be expressed as

$$\Delta p = -\rho u^2(t/R)$$

The bubble pressure coefficient is defined as

$$C_{PB} = (p_\infty - p_B)/\rho u^2 = -t/R \qquad (2)$$

Expressing the jet centerline radius of curvature R as

$$R = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$$

Substituting Eqs. (1) and (2) and integrating gives the jet centerline slope as

$$\frac{dy}{dx} = -\left\{\frac{1}{[[C_{PB}/(t_i(t_i + D))]\cdot[y^2 - ((t_i + D)/2)^2] + 1]^2} - 1\right\}^{1/2} \qquad (3)$$

The boundary conditions at $x = 0$ are

a) $y = (t_i + D)/2$ b) $dy/dx = 0$

Equation (3) is an elliptic function and when integrated gives the following expression for the axial location of the curved jet centerline

$$x = [2t_i(t_i + D)/C_{PB}]^{1/2} \cdot E(\phi, k) - [t_i(t_i + D)/2C_{PB}]^{1/2} \cdot F(\phi, k) \qquad (4)$$

where E and F are elliptic integrals of the first and second kind, respectively. The amplitude ϕ and modulus k are

$$\phi = \cos^{-1}[2y/(t_i + D)] \qquad (4a)$$

and

$$k = [C_{PB}(t_i + D)/8t_i]^{1/2} \qquad (4b)$$

From the geometry of the flow shown in Fig. 1, this inviscid solution for the reattachment length gives

$$x_R = x_c - y_c \tan\theta_c \qquad (5)$$

where

$$y_c = \tfrac{1}{2}[t_i(t_i + D)\cos\theta_c]^{1/2} \qquad (5a)$$

and

$$\cos\theta_c = \frac{(C_{PB}/4)(1 - D/t_i) - 1}{(C_{PB}/4) - 1} \qquad (5b)$$

Solving Eqs. (4) and (5) for arbitrary values of the bubble pressure coefficient and jet thickness ratio gives the inviscid solution for the reattachment length. Figure 2 shows this solution in nondimensional form.

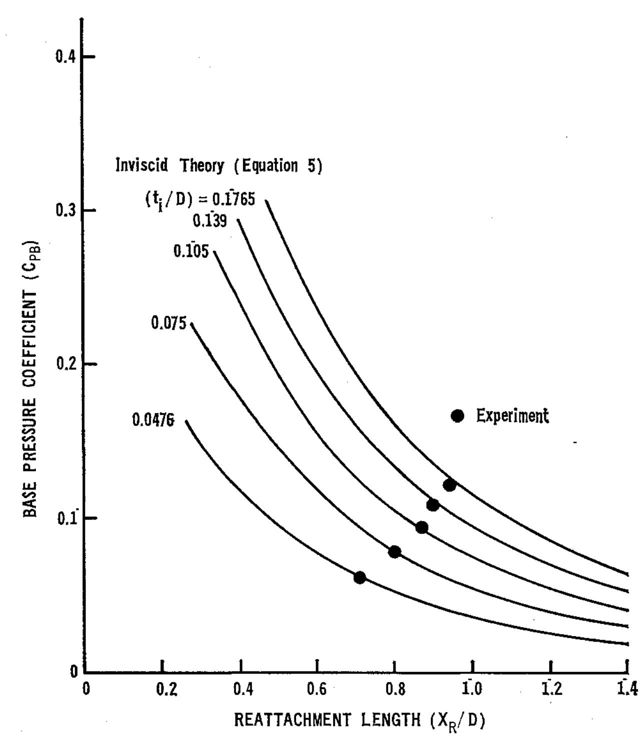

Fig. 2 Relationship between pressure in base region and position of jet reattachment.

Base Drag

To make the inviscid solution of reattachment length unique, flow entrainment is now included in a momentum balance for purposes of evaluating the over-all jet thrust recovery, the base pressure coefficient and the recirculation mass flow. Considering the analytic model outlines in Fig. 1, the net lifting force J_o is

$$J_o = I_i - (p_\infty - p_B)A_B + J_R \tag{6}$$

J_o is to be determined sufficiently far downstream of the reattachment point that the pressure is atmospheric across the jet. The second term on the right-hand side of Eq. (6) is the pressure force due to the base bubble pressure (which has been assumed constant) and the third term is the recirculating mass flow impacting on the base. Together these two terms allow for one pressure difference to act across the jet and a lesser "effective pressure" to act on the solid base.

Considering only entrainment on the inner surface of the reattaching jet (i.e. $\dot{m}_o = \dot{m}_i$), Eq. (6) can be rearranged to give

$$\frac{u_o}{u_i} = \frac{1 - C_{PB}(A_B/A_J + 1/2)}{1 - \dot{m}_R/\dot{m}_i} \tag{7}$$

A second expression for the velocity ratio can be written from mass conservation and assumptions 5 and 7

$$u(\dot{m}_i + \dot{m}_R) = u\dot{m}_o + u\dot{m}_R = u_i \dot{m}_i$$

Combining this with Eq. (7) gives

$$u_o/u_i = 1/(1 + \dot{m}_R/\dot{m}_i) \tag{8}$$

Expressing the recirculation mass flow ratio in a manner similar to flow entrainment for plane jets[9] gives

$$\frac{\dot{m}_R}{\dot{m}_i} \propto \left[\frac{s}{t_i}\right]^{1/2}$$

Approximating the distance s as the length of the side of a cone of base diameter D and height x_R gives

$$\dot{m}_R/\dot{m}_i = \sigma[(x_R/t_i)^2 + (D/2t_i)^2]^{1/4} \tag{9}$$

The constant of proportionality, σ, becomes the mixing coefficient and is to be evaluated experimentally.

The thick annular jet solution for reattachment length is now complete. The inviscid solution, Eqs. (4) and (5) gives the reattachment length (x_R/D) as a function of bubble pressure coefficient (C_{PB}) and initial jet geometry (t_i/D). The viscous solution, Eqs. (7, 8 and 9) can be solved for the mixing coefficient (σ) as a function of reattachment length and initial jet geometry. Together, the inviscid and viscous solutions can be solved for σ as a function of x_R/D and t_i/D. Plotting experimentally determined values of x_R/D vs σ then gives the appropriate value of σ. Once σ is established, the reattachment length, the base pressure coefficient and recirculation mass flow ratio can be readily established for any annular jet thickness ratio.

A convenient index of the total performance losses for a thick annular jet (i.e., the base drag) is the thrust recovery factor which is defined as

$$\eta = J_o/I_i$$

From Eq. (6), the recovery factor can be expressed as

$$\eta = 1 - \frac{1}{1 - (C_{PB}/2)}\left[C_{PB}\frac{A_B}{A_J} - \frac{1}{(\dot{m}_i/\dot{m}_R) + 1}\right] \tag{10}$$

Experimental Investigation

Air was supplied to the model by a 700 cfm blower through a 55 gallon drum plenum chamber and a three in. drawn brass inlet tube (2.76 in. ID × 30 diam. long). A 24-in. diam. plastic disc, supported on adjustable arms so that it could be moved along the axis of symmetry only, acted as the thrust recovery measuring ground board. The model head outside diameter was fixed (5.75 in.), and the jet size was varied by changing center pieces. Five jet thicknesses ($t_i/D = 0.0476$, 0.0750, 0.1053, 0.1389, and 0.1765) were tested. Jet Reynolds numbers (ut_i/ν) were of the order of 2×10^4.

Static pressure profiles along the axis of symmetry were made by inserting a long, spherical tip static pressure tube through the center of the ground board. Measurements were made at 0.25 in. intervals from the model base to the ground board.

The ground board was fitted with numerous static pressure taps in several radial directions. Six of these, located at a constant radius, were used to check flow symmetry. In all cases flow symmetry was good. The ground board was mounted four base diameters downstream and fitted with strain gauges to measure thrust recovery.

The inlet pipe was instrumented with a cylindrical head total pressure tube aligned along its centerline and a wall static pressure tap. Both were 26 pipe diameters downstream from the plenum chamber. In all experiments the mass flow rate was established from calibrated inlet pipe measurements. The jet exit total and static pressure readings were taken with a cylindrical head total pressure tube and a spherical head static pressure tube (0.006 in. ID × 0.030 in. OD with four 0.009 in. holes spaced at 90° located 10 diam. from the tip), respectively. Jet exit measurements were taken at $\frac{1}{8}$ in. intervals across the jet exit plane. Mass flow rates were calculated from both the jet exit and the inlet tube pressure measurements as a check of the jet exit measurements.

Discussion of Experimental Results and Comparison with Theory

The Mixing Coefficient

The mixing coefficient in Eq. (9) was evaluated by plotting experimentally determined values of the reattachment length vs the jet thickness ratio and comparing these results to the values predicted from the theory for several values of σ. The reattachment length was assumed to be located at the position of the peak axial static pressure (Fig. 3) as determined by the axial pressure measurements. The experimental points fall into a relative narrow band of σ values between 0.040 and 0.050. A value of $\sigma = 0.046$ was selected

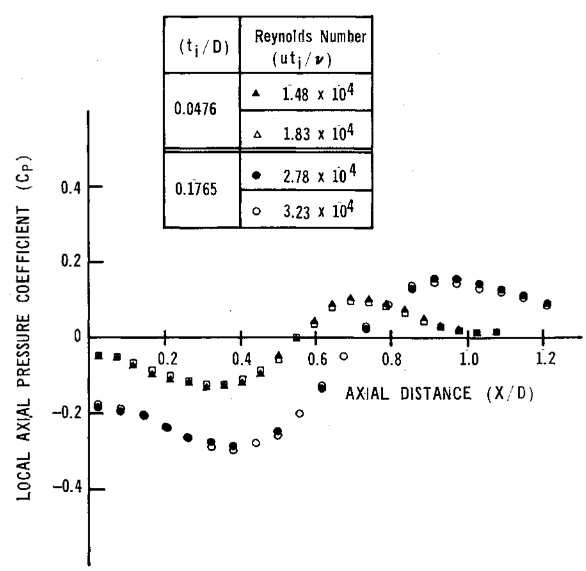

Fig. 3 Centerline pressure profiles.

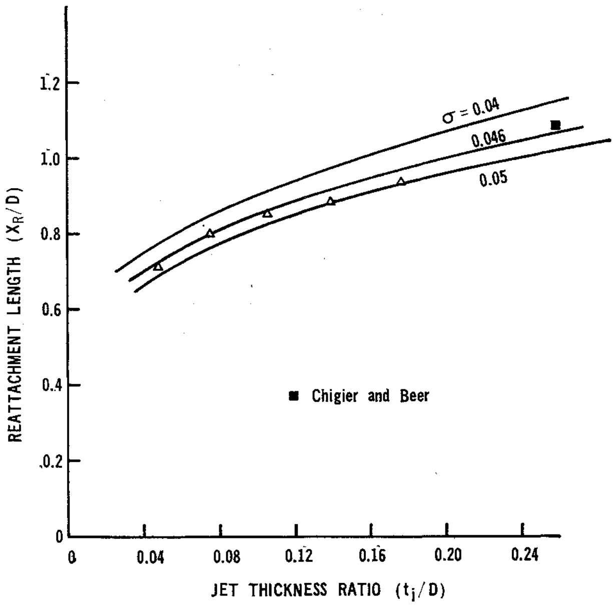

Fig. 4 Experimental determination of appropriate jet mixing coefficient.

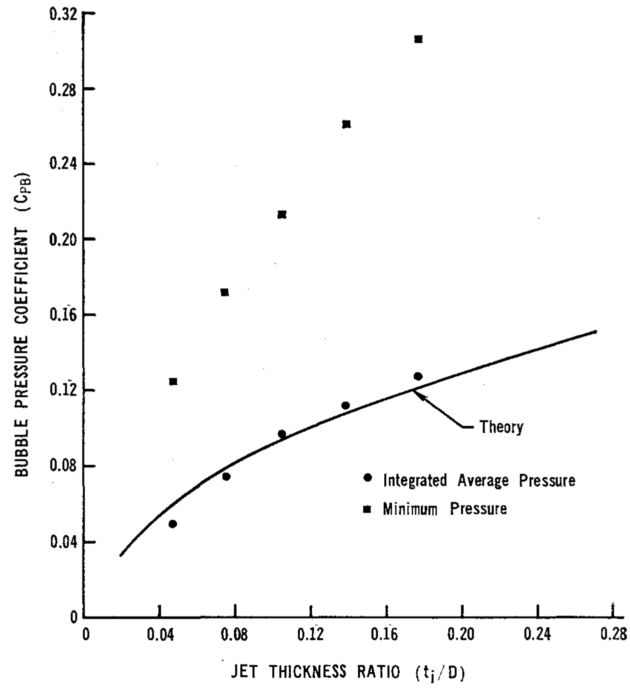

Fig. 5 Base pressure coefficient.

as the mixing coefficient giving the best over-all agreement. A point taken from Chigier and Beer is included on Fig. 4 and shows good agreement.

As noted by Bourque and Newman, the exact location of the reattachment point is difficult to define with measurements of this type. With entraining jets, the smoothing of the velocity profile may actually locate the stagnation point slightly upstream of the pressure peak. This effect should be minimized as the jets become thicker (decreasing s/t_i at reattachment).

Bubble Pressure Coefficient

Typical axial static pressure distributions, as measured by the cylindrical head probe, are shown in Fig. 3. The pressure rise along the axis from the point of minimum bubble pressure to the model base is indicative of the recirculation mass flow stagnating on the base and thus being "recovered" as a thrust.

The bubble pressure coefficients predicted by theory are shown in Fig. 5 as a function of jet thickness ratio. Also shown are the integrated average values of the centerline pressure coefficient from Fig. 3. This integration was of the form

$$C_{PB} = \frac{1}{x_R/D} \int_0^{x_R/D} C_P(x) d(x/D)$$

and was evaluated graphically. The agreement with theory is quite good and demonstrates that the average centerline pressure is representative of the theoretical base bubble pressure. Bourque and Newman found that for their two-dimensional offset jet case, the minimum pressure measured along the wall in the recirculation region was approximately equal to the mean bubble pressure as determined by their pressure traverses. For the three-dimensional annular jet considered here, the minimum axial pressures are observed to be much larger than the mean pressure.

Recirculation Mass Flow Ratio

Theoretical values for the recirculation mass flow are shown in Fig. 6 and compared with an experimental result from Chigier and Beer. This figure illustrates the tendency

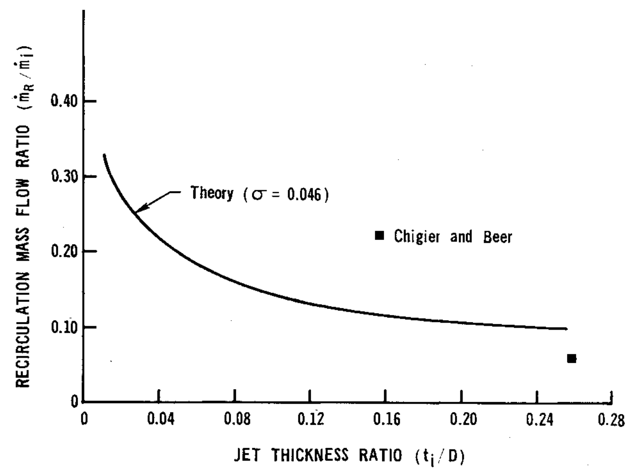

Fig. 6 Recirculation mass-flow within base region.

for large internal entrainment ratios ($\dot{m}_R/\dot{m}_i$) as the jet thickness ratio (t_i/D) decreases. It can therefore be expected that the theoretical model proposed in this study will show less agreement with experiment as the annular jet becomes comparatively thin.

Thick Annular Jet Base Drag or Thrust Recovery

Figure 7 shows the predicted thrust recovery compared with experiment for several jet thicknesses. The experimental values were determined by computing the jet impulse at the nozzle exit from measured total and static pressures and comparing this to the force on the ground board measured by the strain gauges. Included on this figure are values taken from Chaplin[4] for the mixing constant $K_2 = 5.0$. The theory developed here (Eq. 10) shows relatively good agreement with the experiment, except for small jet thickness to base diameter ratios. The lack of agreement for thin jets results from a violation of assumptions 4 and 6 within the

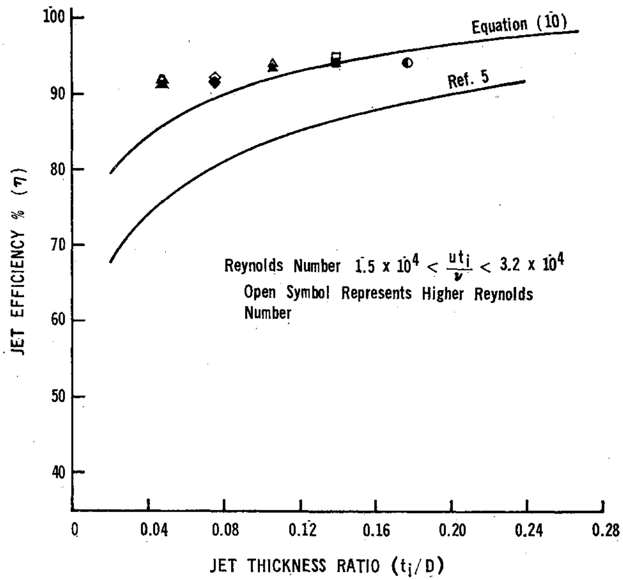

Fig. 7 Jet thrust recovery.

theory. For thin jets, jet entrainment cannot be neglected and the applicability of the inviscid solution is doubtful. This results in a higher theoretical base pressure coefficient and thus a lower thrust recovery.

When considering the efficiency of thrust recovery, a trade-off exists between the base drag due to the negative pressure acting on the base area and the recovery due to the impacting recirculation flow. It can be shown that the recirculation flow acts to recover approximately 50% of the base pressure loss for a thickness ratio of $t_i/D = 0.04$ and about 75% of the loss for $t_i/D = 0.2$.

Conclusions

The base drag, reattachment length and base pressure coefficient for a thick annular jet are expressible as functions of the jet thickness ratio. The theory developed in this analysis adequately predicts these parameters. The results support the assumptions made in the analysis for moderately large jet thickness to base diameters ratios.

The reattachment length and the base pressure coefficient increase with increasing jet thickness ratio while the recirculation mass flow ratio and base drag decrease. The base drag or thrust loss is less than 10% of the initial jet impulse for jet thickness ratios of 0.048 or larger.

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